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EVALUATING THE STABILITY OF ESTIMATORS IN NON-GAUSSIAN AUTOREGRESSIVE MODELS WITH OUTLIERS UTILIZING POSITIVELY SKEWED DISTRIBUTION

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ABSTRACT

The research work seeks to find out the efficiency of some methods of estimation of autoregressive model where the underlying distribution is positively skewed. To examine the performance of estimators on positively skewed data, data were simulated from various distributions (Weibull, Beta, Gamma, and Exponential distribution) along different sample sizes. Outliers are one of the major problems affecting probability distributions and methods of estimations. Outlier points can therefore indicate faulty data, erroneous procedures, or areas where a certain theory might not be valid. In line with the objectives of this work, outliers were injected at different percentage (10%, 25% and 50%) in every stage of the simulation process. Data fitted were estimated using Ordinary Least Square, Maximum Likelihood, BURG and Yule-walker methods of estimation to compare the efficiency of these estimators. Data simulated were from sample sizes n=5, n=10, n=25, n=50, n=100, n=200, n=500, n=1000, n=2000 and n=5000 were fitted to check the consistencies of the aforementioned estimators. The shape and scale parameter of Weibull, Gamma, and Beta distribution were varied at 2 and 1, to check the pattern in which the estimated results vary. The performance of Beta distribution is better in all the sample sizes irrespective of the orders being used. Order 3 and 4 under MLE and order 2,3,4 under OLS have the best estimates while the other sample sizes have small estimates and are the same. Beta distribution performs better than other distributions. The shape and scale parameter of Beta have little or no effect on the distribution itself.

Keywords: Autoregressive models, Non-Guassian, Outliers, Simulation, Estimation

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INTRODUCTION

Time series modelling is a dynamic research area that has attracted the attention of the research community over the last few decades. The main aim of time series modelling is to carefully collect and rigorously study the past observations of a time series to develop an appropriate model that describes the inherent structure of the series. This model is then used to generate future values for the series, i.e., to make forecasts. Time series forecasting can thus be termed the act of predicting the future by understanding the past[1]. Due to the indispensable importance of time series forecasting in numerous practical fields such as business, economics, finance, science and engineering, etc.[2], proper care should be taken to fit an adequate model to the underlying time series. It is obvious that successful time series forecasting depends on an appropriate model fit. Over many years, researchers have made a lot of efforts to develop efficient models to improve forecasting accuracy. As a result, various important time series forecasting models have evolved in the literature. One of the most popular and frequently used stochastic time series models is the Autoregressive Integrated Moving Average (ARIMA)[3]. The basic assumption made to implement this model is that the considered time series is linear and follows a particular known statistical distribution, such as the normal distribution. The ARIMA model has subclasses of other models, such as the autoregressive (AR)[4], moving average (MA)[5] and autoregressive moving average (ARMA)[6] models.

For seasonal time series forecasting, Afrifa-Yamoah *et al.* 2016 had proposed a quite successful variation of the ARIMA model, viz. the Seasonal ARIMA (SARIMA)[7]. The popularity of the ARIMA model is mainly due to its flexibility to represent several varieties of time series with simplicity, as well as the associated Box-Jenkins methodology for an optimal model-building process[8.] But the severe limitation of these models is the pre-assumed linear form of the associated time series, which becomes inadequate in many practical situations. To overcome this drawback, various non-linear stochastic models have been proposed in the literature[9]; however, from an implementation point of view, these are not as straight-forward and simple as the ARIMA models.

Autoregressive models are a class of specifications where one attempts to model and predict financial variables using only information contained in their own past values and possibly current and past values of observation. Indeed, it is a time series model; unlike moving average and autoregressive moving average models, it deals with only observed values. Time series models are usually a-theoretical, implying that their construction and use are not based upon any underlying theoretical model of the behaviour of a variable. Instead, time series models are an attempt to capture empirically relevant features of the observed data that may have arisen from a variety of different (but unspecified) structural models. An important class of time series models is the family of autoregressive integrated moving average (ARIMA) models, usually associated with. The estimation of coefficients in a simple auto-regressive model is an important problem and has received a great deal of attention in the literature. Most of the work reported is, however, based on the assumption of normality[10-12] They assumed normality but based their estimators on censored samples. They showed that the resulting estimators are robust to plausible deviations from normality. In recent years, however, it has been recognised that the underlying distribution is, in most situations, basically non-normal Beta and Gamma,[13.]

The problem, therefore, is to develop efficient estimators of coefficients in autoregressive models when the underlying distribution is non-normal. The distributions to be considered are only positively skewed continuous distributions like lognormal, exponential, and gamma. Their performance will be compared with the positively skewed normal distribution[14]. Naturally, one would prefer the best estimators, which are fully efficient. Preferably, these estimators should also be robust to plausible deviations from an assumed model. [2.15] studied the estimation in autoregressive models with the underlying distribution being a shift-scaled Student's t distribution. They developed the modified maximum likelihood (MML) estimators of the parameters and showed that the proposed estimators had closed forms and were remarkably efficient and robust.

Outliers are a frequent issue in autoregressive regression models, as they have negative impacts on the least squares estimators. To solve this issue, numerous regression estimation strategies have been proposed. The majority of these methods are extensions of the traditional least squares method. In the regression scenario, a few additional robust strategies have been researched on both theoretical and empirical bases. However, in the context of time series and econometrics, the best estimator(s) of autoregressive models that incorporate a certain percentage of outliers for non-normal data have not drawn much attention[3,6,15] A common problem in autoregressive regression models is outliers, which produce undesirable effects on the least squares estimators. Many regression estimation techniques have been suggested to deal with this problem. The majority of such techniques are developed from the classical least squares. Some other robust approaches have been investigated in the regression case, both on theoretical and empirical grounds. However, the best estimator(s) of autoregressive models that contain some proportion of outliers for non-normal data has not received more attention in the context of time series and econometrics.

A common problem in autoregressive regression models is outliers, which produce undesirable effects on the least squares estimators. Many regression estimation techniques have been suggested to deal with this problem. The majority of such techniques are developed from the classical least squares. An outlier can cause serious problems in statistical analysis. Outliers can occur by chancer4tio in any distribution, but they often indicate either measurement error or that the population has a heavy-tailed distribution. In the former case, one wishes to discard them or use statistics that are robust to outliers, while in the latter case, they indicate that the distribution has high skewness and that one should be very cautious when using tools or intuitions that assume a normal distribution.

Outlier points can therefore indicate faulty data, erroneous procedures, or areas where a certain theory might not be valid. However, in large samples, a small number of outliers is to be expected (and not due to any anomalous condition). Outliers, being the most extreme observations, may include the sample maximum, sample minimum, or both, depending on whether they are extremely high or low. However, the sample maximum and minimum are not always outliers because they may not be unusually far from other observations.

However, to address this problem, this research made use of a non- Gaussian autoregressive model with outliers to get the best estimators that are efficient and consistent across the autoregressive order (1-4).

The aim of this research is to Examine and Analyze the robustness of some estimate methods of autoregressive model where the underline distribution is positively skewed in the presence of outliers using simulation study at different sample sizes. The specific objectives are to:

1. examine the properties of these estimators when a proportion of outliers are introduced in the samples
2. propose a suitable estimator for non-Gaussian Autoregressive model with outliers
3. Analyze the effect of outliers on the estimators
4. Determine the best estimator at various sample sizes and distributions

Methods of estimation

We need to only concern ourselves with the problem of estimating the parameters in autoregressive models. In practice, then we treat the *p*th difference of the original time series as the time series from which we estimate the parameters of the complete model. For simplicity, we shall let $\alpha_0, \alpha_1, \ldots, \alpha_p$denote our observed autoregressive process even though it may be an appropriate difference of the original series. We first discuss the method-of maximum likelihood estimator, and least squares estimator.

Maximum Likelihood Estimation

For any set of observations, Y_1, $Y_2,\ldots,Y_n$, time series or not, the likelihood function L is defined to be the joint probability density of obtaining the data actually observed. However, it is considered as a function of the unknown parameters in the model with the observed data held fixed. For ARIMA models, L will be a function of the α's, θ's, μ, and σ_e^2given the observations Y_1, $Y_2,\ldots,Y_n$. The maximum likelihood estimators are then defined as those values of the parameters for which the data actually observed are most likely, that is, the values that maximize the likelihood function [16].

We begin by looking in detail at the AR[1] model. The most common assumption is that the white noise terms are independent, normally distributed random variables with zero means and common standard deviationσ_e . The probability density function (pdf) of each e_t is then

$$(2\pi\sigma_e^2)^{-\frac{1}{2}} \exp\left(-\frac{e_t^2}{2\sigma_e^2}\right) for - \infty < e_t \infty$$

and, by independence, the joint pdf for e_2, e_3…, e_n is

$$(2\pi\sigma_e^2)^{-(n-1)/2} \exp\left(-\frac{1}{2\sigma_e^2}\sum_{t=2}^{n} e_t^2\right)$$

Auto Regressive Models of Order P

AR[1]=$\alpha_0 + \alpha_1 Y_{t-1} + e^t$

AR[2]=$\alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + e^t$

.AR(P)=$\alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \cdots + \alpha_p Y_{t-p} + e^t$

Ordinary Least squares

Consider the first-order case where

$$Y_t - \mu = \emptyset(Y_{t-1} - \mu) + e_t$$

We can view this as a regression model with predictor variable Y_{t-1} and response variable Y_t. Least squares estimation then proceeds by minimizing the sum of squares of the differences $(Y_t - \mu)$–$\emptyset(Y_{t-1} - \mu)$

Since only Y_1, Y_2,…,$Y_n$are observed, we can only sum from t = 2 to t = n. Let

$$S_c(\emptyset, \mu) = \sum_{t=2}^{n}[(Y_t - \mu) - \emptyset(Y_{t-1} - \mu)]^2$$

Taking$\frac{\partial S_c}{\partial \mu} = 0$, we have

$$\frac{\partial S_c}{\partial \mu} = \sum_{t=2}^{n} 2[(Y_t - \mu) - \emptyset(Y_{t-1} - \mu)](-1 + \emptyset) = 0$$

When simplifying and solving

$$\hat{\mu} = \frac{1}{(n-1)(1-\emptyset)}\left[\sum_{t=2}^{n} Y_t - \emptyset \sum_{t=2}^{n} Y_{t-1}\right]$$

Similarly when differentiating with respect to $\emptyset$ we have

$$\frac{\partial S_c(\emptyset,\bar{Y})}{\partial \emptyset} = \sum_{t=2}^{n} 2[(Y_t - \bar{Y}) - \emptyset(Y_{t-1} - \bar{Y})](Y_{t-1} - \bar{Y}) = 0$$

$$\hat{\emptyset} = \frac{\sum_{t=2}^{n}(Y_t - \bar{Y})(Y_{t-1} - \bar{Y})}{\sum_{t=2}^{n}(Y_{t-1} - \bar{Y})^2}$$

Following the same procedure we can obtain estimation of parameters for second and higher orders of autoregressive model [17] for detail of estimations.

Yule Walker Method

The Yule-Walker Method block estimates the power spectral density (PSD) of the input using the Yule-Walker AR method. This method, also called the autocorrelation method, fits an autoregressive (AR) model to the windowed input data by minimizing the forward prediction error in the least-squares sense. This formulation leads to the Yule-Walker equations, which are solved by Levinson-Durbin recursion[17].

Burg Method

The Burg Method block estimates the power spectral density (PSD) of the input frame using the Burg method. This method fits an autoregressive (AR) model to the signal by minimizing (least-squares) the forward and backward prediction errors while constraining the AR parameters to satisfy the Levinson-Durbin recursion.

Distributions Considered

Weibull Distribution

A random variable x has a weibull distribution if and only if the probability density is given by

$$f(x) = \begin{cases} Kx^{\beta-1} & e^{-\alpha x^{\beta}} \qquad for\ x > 0 \\ 0\ elsewhere & \end{cases}$$

Gamma Model.

A random variable x has a gamma distribution, and it is referred to as a gamma random variable if and only if its probability density is given by

$$f(x) = \begin{cases} \frac{x^{\alpha-1}e^{-\frac{\alpha}{\beta}}}{\beta^{\alpha}r(\alpha)} & for\ x > 0 \\ 0\ \ elsewhere & \end{cases}$$

Where α>0 and β>0.

The gamma AR(1) process(X_t), with gamma ($\beta(1-\alpha), v$) Marginal distribution was constructed by Sim (1990) as

$X_t = \alpha * X_{t-1} + \varepsilon_t$

Where (ε_t) is a sequence of IID gamma (β, v) random variables with $\beta, v > 0$and the operator '$*$' is defined as in model[2]. The marginal density of (X_t) and its conditional density are respectively,

$f_x(x) = [\beta(1-\alpha)]^{v} x^{v-1} \exp[-\beta(1-\alpha)x] / \Gamma(v)$and

$f X_{i+j}/X_i(x/y) = \theta\left(\frac{x}{\alpha^j y}\right)^{\frac{(v-1)}{2}} \exp[-\theta(x + \alpha^j y)] I_{v-1}[2\theta(\alpha^j xy)^{\frac{1}{2}}]$

where$\theta = \beta(1\text{-}\alpha)/(1\text{-}\alpha^j)$,0<$\alpha$ <1, and $I_r(z)$ is the modified Bessel function of the first kind and of order r. Another well-developed gamma model is the GAR[1]model of [17]. The GAR[1] model was constructed from the simple difference equation[1]

Beta Distribution

A random variable x has a beta distribution and it is referred to as a beta random variable if and only if its probability density is given by

$$f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)x^{\alpha-1}(1-x)^{\beta-1}}{\Gamma(\alpha)\Gamma(\beta)} & for\ 0 < x < 1 \\ 0\ \ elsewhere & \end{cases}$$

Where α>0 and β>0.

Exponential Model

A random variable x has an exponential distribution and it is referred to as an exponential random variable if and only if its probability density is given by

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & for\ x > 0 \\ 0\ elsewhere & \end{cases}$$

Where θ>0

By assuming that the process [Xt] has an exponential marginal PDF with parameter X_t, Gaver and Lewis (1980) showed that the innovation process ε_t of model (1) take the form

$$\varepsilon_t = \begin{cases} 0 & \text{with probability } \alpha \\ E_t & \text{with probability } 1-\alpha \end{cases}$$

where 0<α <1 and (E_t)is a sequence of IID exponential random variables with parameter ⋋. The conditional density of X_{i+j} given $X_t = yis$

$$fX_{i+j}/X_i(X/Y) = \alpha^j\delta(x-\alpha^j y) + ⋋ (1 - \alpha^j)\exp[- ⋋ (x-\alpha^j y)] \cup (x - \alpha^j y),$$

$-\infty < x,\ y < \infty$

whereδ(x) is the Dirac delta function and U(u) is the unit step function. This exponential AR [1]model has been generalized by Golubev[6.] to the two-parameter NEAR [1] model and the three-parameter GEAR[1] model, respectively. Both models have a tractable joint PDF and both are likely candidates for our model-building approach.

Design of Simulation Studies

Simulation studies use computer intensive procedures to test particular hypotheses and assess the appropriateness and accuracy of a variety of statistical methods in relation to the known truth. These techniques provide empirical estimation of the sampling distribution of the parameters of interest that could not be achieved from a single study and enable the estimation of accuracy measures, such as the bias in the estimates of interest, as the truth is known (Kim *et al.*, 2014). Simulation studies are increasingly being used in the medical literature for a wide variety of situations. In addition, simulations can be used as instructional tools to help with the understanding of many statistical concepts [18]. Designing high quality simulations that reflect the complex situations seen in practice, such as in randomized controlled trials or prognostic factor studies, is not a simple process. Simulation studies should be designed with similar rigour to any real data study, since the results are expected to represent the results of simultaneously performing many real studies. Unfortunately, in very few published simulation studies are sufficient details provided to assess the integrity of the study design or allow readers to understand fully all the processes required when designing their own simulation study. Performing any simulation study should involve careful consideration of all design aspects of the study prior to commencement of the study from establishing the aims of the study, the procedures for performing and analyzing the simulation study through to the presentation of any results obtained. These are generic issues that should be considered irrespective of the type of simulation study but there may also be further criteria specific to the area of interest, for example survival data. It is important for researchers to know the criteria for designing a good quality simulation study. The aim of this is to provide a comprehensive evaluation of the generic issues to consider when performing any simulation study, together with a simple checklist for researchers to follow to help improve the design, conduct and reporting of future simulation studies. The basic concepts underpinning the important considerations will be discussed, but full technical details are not provided and the readers are directed towards the literature [2,18]. General considerations are addressed rather than the specific considerations for particular situations where simulations are extremely useful, such as in Bayesian clinical trials design[3] sample size determination[16] or in studies of missing data. A small formal review of the current practice within published simulation studies is also presented

Simulation Study

The following distributions were used in simulating:

(i) Weibul Distribution
(ii) Gamma Distribution
(iii) Beta Distribution
(iv) Exponential Distribution

Sample sizes n of 5, 10, 20, 50, 100, 200, 500, 1000, 2000 and 5000 were considered to account for small sizes Mild sizes and large Sizes. Furthermore, the following methods of estimation were considered in relative to their respective orders varying from order 1 to order 4

Data Analysis

R-package was used in simulating data with sample size ranging from n=5, 10, 25, 50, 100, 200, 500, 1000, 2000 and 5000. Order 1 to 4 were analysed. The tables below show the summary result of the analysis.

The table below indicates the best distribution and sample sizes of order one to four estimate for all estimators with 25% outliers. The table explains that using the method of AIC, the best distribution is the Beta distribution and the estimator method is maximum likelihood estimate at order one to four and a sample size of $n = 50$.

Summary of Simulated Data with Twenty-Five (25) Percent Outliers

Sample size n	Beta Order 1 MLE	Beta Order 2 MLE	Beta Order 3 MLE	Beta Order 4 MLE
30	0.07006	0.06719	0.06719	0.06719
50	0.04674	0.04674	0.04674	0.04674
100	0.06483	0.06206	0.06016	0.05882
500	0.07539	0.07269	0.06945	0.06921
1000	0.06619	0.06213	0.06079	0.05953
2000	0.06837	0.0661	0.06414	0.06289
5000	0.06821	0.06596	0.06444	0.06289

Result of Simulated Data with Fifty (50) Percent Outliers

Sample size n	Beta order 1 OLS	Beta order 2 OLS	Beta Order 3 OLS	Beta Order 4 OLS
30	0.06064	0.06064	0.06064	0.06064
50	0.05724	0.05375	0.0535	0.04822
100	0.07136	0.06538	0.06133	0.06037
500	0.0787	0.7474	0.06988	0.06936
1000	0.07065	0.06599	0.06406	0.06255
2000	0.07215	0.06911	0.06661	0.06479
5000	0.07175	0.06825	0.06584	0.064579

The table above indicates the best distribution and sample sizes of order one estimate for all estimators with 50% outliers. The table explains that using the method of AIC, the best distribution is the Beta distribution, with Ordinary least square with a sample size of $\mathrm{n} = 50$ as it have the lowest value at order

Best DISTRIBUTION estimate across sample sizes and Orders without outliers

SAMPLE SIZE	OLS ORDER 1	OLS ORDER 2	OLS ORDER 3	OLS ORDER 4
N= 30				
N=50	0.07428	0.07428	0.07428	0.07428
N=100	0.06341	0.06341	0.06341	0.06341
N=500	0.06318	0.06318	0.06318	0.06318
N=1000	0.06047	0.06047	0.06047	0.06047
N=2000	0.05778	0.05778	0.05778	0.05778
N=5000	0.05511	0.05511	0.05511	0.05511

The key observation from the table is that as the sample size increases, the Ordinary Least Square (OLS) values become more consistent across the different model orders (1, 2, 3, and 4). This suggests that with larger sample sizes, the choice of model order may have a smaller impact on the OLS estimates, and the model selection can be more flexible. However by method of AIC the best distribution is the Beta distribution with the ordinary least square estimate at order one to four and sample size $n = 5000$

Conclusion

The performance of Beta distribution is better in all the sample sizes irrespective of the orders being used. Order 3 and 4 under MLE and order 2,3,4 under OLS have the best estimates while the other sample sizes have small estimates and are the same.

Beta distribution perform better than other distributions. The shape and scale parameter of Beta have little or no effect on the distribution itself. The result from the outlier inclusion showed that beta distribution was consistent irrespective of the distribution, sample sizes and the order of Autoregressive model. At sample size 100, the estimate of BURG and MLE are the same. At sizes 500 -5000 the estimates are approximately the same.

Recommendations

i) Beta distribution could be given more attention when considering a non-Gaussian situation.

ii) Beta distribution could be used in place of Gaussian Distributions because it gives a better estimate even with outliers.

iii) Beta distribution gives a robust result with 20%, 25% and 50% outliers therefore it is recommended as a better distribution in autoregressive model.

REFERENCES

1.Adesanya, K. K., Taiwo A. I, Adedotun A. F. and Olatayo T. O. Modeling Continuous Non-Linear Data with Lagged Fractional Polynomial Regression. Asian Journal of Applied Sciences,2018, 5, 315-320.

2. Adedotun, A. F., Taiwo, A. I and Olatayo, T. O. A Re-parametrised Autoregressive Model for Modelling Gross Domestic Product. *ANNALS of Faculty Engineering Hunedoara-Int'l J of Engir*, 2020, 17 (2), 184-188

3. Adhikari, R. and Agrawal, A. An Introductory Study on Time Series Modeling and Forecasting. *LAP Lambert Academic Publishing*, 2013,5 (2), pp. 1302-1401.

4. Afrifa-Yamoah, E., Bashiru, I.I., Saeed, R. and Karim, A. Sarima Modelling and Forecasting of Monthly Rainfall in the Brong Ahafo Region of Ghana. *World Environment*, 2016, 6(1), 1-9.

5. Angelis, G. I., Gillam, J. E., Ryder, W. J., Fulton, R. R. and Meikle S. R. Direct estimation of voxel-wise neurotransmitter response maps from dynamic pet data. *IEEE Trans. Med. Imaging 2019*, 38, 1371–1383.

6. Balyani, Y., Niya, G.F., and Bayaat, A. A study and prediction of annual temperature in Shiraz using *ARIMA model. J. Geographic Space*, 2014, 12(38), 127-144.

7. Doualh, M.S. Time Series Forecasting: A comparative Study of VAR ANN and SVM Models. *J of Statical and Econometric Methods,2019*, 8(3), 21-34.

8.Howard, C.C., Howard, I. C. and Etuk, E. H. Time Series Modeling and Prediction a

Review. *Intl J of Afri Sustainable Devt*, 2019,10 (2), pp. 38-39.

9. Kruschke, J. K. Doing Bayesian Data Analysis: A Tutorial with R, JAGS and Stan. Academic Press / Elsevier. 2015, ISBN 978-0-12-405888-0.

10. Li, J., Shao, F. and Wing-Keung, W. (2015). Estimating Parameters in Autoregressive Models with Asymmetric Innovations: MMLE and Nonlinear Approaches. Social Science Research Network,2015, 9 (3), pp. 40-46.

11. Kim, S. J., Sullivan, J. M., Wang, S., Cosgrove, K. P. and Morris, E. D. Voxelwise lp- ntPET for detecting localized, transient dopamine release of unknown timing: sensitivity analysis and application to cigarette smoking in the pet scanner. *Hum. Brain Mapp*.2014, 35, 4876–4891. 10.1002/hbm.22519 [

12. Stoffer, D. and Poison, N. Applied Statistical Time Series Analysis, ASTSA,2023, 5 (1) pp. 45-50

13. Thomas, N., Franzke C. L. E. and Vannacvi A. Predictability and Non-Gaussian Characteristics of the North Atlantic Oscillation. *American Meteorological Society*,2017, 9(2), 589-602.

14. Tealab, A. Time series forecasting using artificial neural network methodologies: A systematic review. *Future Computing and Informatics J ,2018,* 3 (1), pp. 334-340.

15. Xu, Z. Modeling Non-Gaussian Time-correlated Data Using Nonparametric Bayesian Method. Ph.D. Thesis Ohio State University.2014, Pp. 40-44.

16. Wooldridge, J. Introductory Econometrics: A Modern Approach (5 ed.). Mason, OH: South-Western Cengage Learning. 2013, Pp. 44.

17. Zare, F., Zare, H. and Fallahnezhad, M. S. Software effort estimation based on the optimal Bayesian belief network. *Applied Soft Computing J* 49, 968–980. https://doi.org/10.1016/j.asoc.2016.08.004

18. Z hang, Y. and Zhang, H. Application of Random Walks in Data Processing. *Highlights in Science, Engineering and Technology*, 2020, 31 (3), pp.45-50.